Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 18

B.Tech. (ME) (2012 Onwards) (Sem.–5)

MATHEMATICS-III

Subject Code : BTAM-500

M.Code : 70601

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt ANY FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt ANY TWO questions.**

SECTION-A

Write briefly :

1. Expand $f(x) = |\sin x|$ in Fourier series.

2. Find Laplace transform of $\sinh t \cos^2 t$.

3. Find Laplace transform of $\dfrac{e^{-at} - e^{-bt}}{t}$.

4. Find inverse Laplace transform of $\dfrac{e^{-7s}}{(s+3)^3}$

5. Express $x^4 + 2x^3 - 6x^2 + 5x - 3$ in terms of Legendre polynomials.

6. For Legendre polynomial $P_n(x)$, show that $P_n'(1) = \dfrac{n(n+1)}{2}$

7. Form a partial differential equation by eliminating arbitrary functions from the relation $z = y f(x) + x g(y)$.

8. Solve $xp + yq = 3z$.

9. Show that the function $f(z) = |z|^4$ satisfies the Cauchy-Riemann equations only at region.

10. State Cauchy Integral Theorem.

SECTION-B

11. Find the Fourier series expansion of the function $f(x) = x^2, -\pi < x < \pi$. Deduce that

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ...$$

12. State and prove Convolution theorem for Laplace transform.

13. For Bessel's function $J_n(x)$, show that $J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + ...) = 1$

14. Solve by Charpit's method $q + xp = p^2$.

15. Evaluate $\int_C \frac{dz}{(z^2+4)^2} = \frac{\pi}{16}, C : |z - i| = 2$

SECTION-C

16. a) Using Laplace transform, solve $y' + 2y = 1 - H(t - 1)$, $y(0) = 2$, where H (t) is Heaviside's unit step function.

b) Find inverse Laplace transform of $\frac{1}{s^2(s+1)}$.

17. a) Using Frobenius method, find two linearly independent solutions of the equation $2x^2y'' + xy' - (x^2 + 1)y = 0$.

b) A rod of length l with insulated side is initially at a uniform temperature u_0. Its ends are suddenly cooled at 0°C and kept at that temperature. Find the temperature function $u(x, t)$.

18. a) Find all Taylor and Laurent series expansions of $f(z) = \frac{1}{z(z-1)}$ about the point z =0.

b) Compute the residues at all the singular points of $f(z) = \frac{z^2}{(z^2+1)^2}$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.